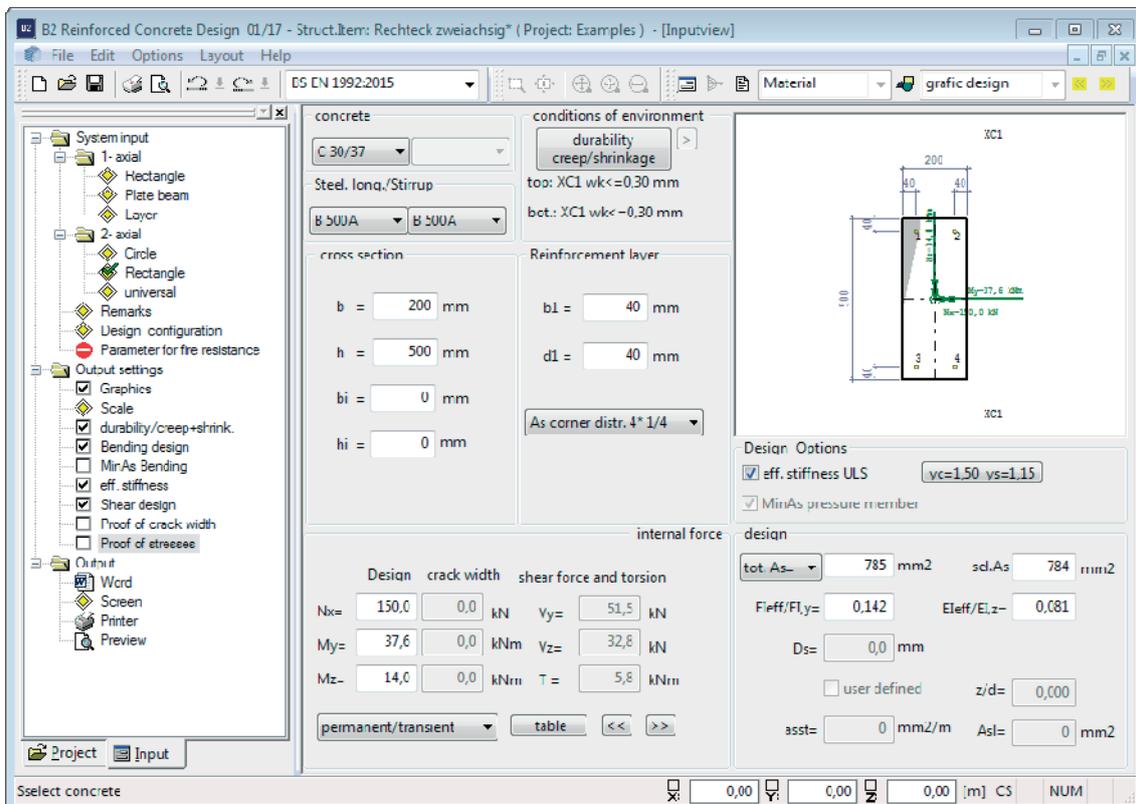


# Analyses on reinforced concrete cross sections

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# Analyses on reinforced concrete cross sections

This document includes additional information about our reinforced concrete software applications.

*Note:* This document describes the Eurocode-specific application. Documents referring to former standards are available in our document archive at [www.friilo.eu](http://www.friilo.eu) Service >> Documentation >>Manuals >>Archive.

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## Standards and acronyms

EN: Recommended values EN 1992-1-1  
EN 1992-1-1:2004 /A1:2014 and EN 1992-1-2:2004 /AC:2008

NDP Parameter defined in the National Annex (NA).

Current versions of the National Annexes (NA):

NA-D: Germany  
DIN 1992-1-1/ NA:2015-09 and DIN EN 1992-1-2/NA:2015-09

NA-A: Austria  
ÖNORM B 1992-1-1:2011 and ÖNORM B 1992-1-2:2011  
These NAs replace those of 2007 applicable recently.

NA-GB: Great Britain  
NA to BS EN 1992-1-1 A2:2015-07, BS8500-1:2015 and NA to BS EN 1992-1-2:2004

NA-I Italy  
UNI EN 1992-1-1/NTC:2008  
and EN 1992-1-2:2004 /AC:2008  
NTC: The application of Eurocode in Italy is described in the "Norme tecniche per le costruzioni" (/ 56 /) and the supplementary circular "Circolare finissima 2.2.2009" (/ 57 /).

NA-PL Poland  
PN EN 1992-1-1:2008/NA:2010 and PN-EN 1992-1-2:2008/NA:2010

## Design for bending and longitudinal force

In the design of reinforced concrete, the strain state causing failure is calculated for the given internal forces while the reinforcement is unknown.

Due to the strain distributions in the ULS defined in the standards, at least one border strain is always known. The internal and external forces must be in balance.

The result is two or, with double bending, three non-linear equations, whereby the internal forces are functions of the border strains and the inclination angle of the neutral axis (double bending). They are resolved by iteration with the help of the Newton method.

You can select among the kh-(kd)-method (only with uniaxial loading) or the method with a given reinforcement ratio for the bending design.

Where cross sections under low loading are concerned, compliance with the minimum reinforcement (compression/bending) can become decisive.

In addition, the application indicates when the permissible maximum reinforcement is exceeded.

### Bases of design

Internal action curve of concrete	Figure 3.3
Maximum strain $f_{cd}$	$\alpha_{cc} \cdot f_{ck} / \gamma_c$
Compressive limit strain of concrete $\varepsilon_{cu}$	$\varepsilon_{cu} = 3.5 \text{ ‰}$ , > C50 irrespective of type of concrete, table 3.1, lightweight concrete, see table 11.3.1
Compressive strain at end of parabolic area $\varepsilon_{c2}$	$\varepsilon_{c2} = 2.0 \text{ ‰}$ , > C50 irrespective of type of concrete, table 3.1, lightweight concrete, see table 11.3.1
Exponent n	$n = 2$ > C50 depending on type of concrete, table 3.1, lightweight concrete, see table 11.3.1
Internal action curve for reinforcing steel	Figure 3.8
Maximum strain $f_{td}$	$K \cdot f_{yk} / \gamma_s$
Limit strain of steel $\varepsilon_{ud}$	NDP
Strain distribution ULS	Figure 6.1

The stress-strain curve of the concrete corresponds to the parabola rectangle stress diagram.

For standard concrete  $\varepsilon_{c2} = 2 \text{ ‰}$  and exponent = 2, closed formulas ( /2/ ) can be used to calculate the internal forces on rectangular or circular cross sections.

In all other cases (high-performance concrete, T-beams and layers cross sections), an approximation calculation is required by splitting the concrete compression zone in thin layers. With cast-in-place complements, the internal forces of the concrete are calculated using the corresponding internal action curves of the different types of concrete used.

You can optionally take the area of the concrete displaced by the steel in the compression zone into consideration (→ B2 [design configuration](#)). The disregard of certain parameters in connection with highly reinforced cross-sections particularly of high-strength concrete, which was common until, recently is no longer justified according to /10/ p. 13.

$f_{ck}$                       Characteristic compressive cylinder strength  
Strength classes acc. to table 3.1

$\alpha_{cc}$  coefficient for long-term effect NDP

NDP	Standard concrete 3.1.6	Lightweight concrete 11.3.5	Unreinforced 12.3.1
EN	1.0	0.85	0.85
NA-D	0.85	0.75	0.75
NA-GB	0.85	= EN	= EN
NA-A	= EN	= EN	= EN
NA-I	0.85	= EN	= EN
NA-PL	= EN	= EN	= EN

$\gamma_c$  partial safety coefficients for concrete NDP

	Permanent/transient 2.4.2.4	Accidental 2.4.3.4	Earthquake
EN	1.5	1.2	1.5
NA-D	= EN	1.3	1.5
NA-GB	= EN	= EN	= EN
NA-A	= EN	= EN	= 1.3
NA-I	= EN	1.0	= EN
NA-PL	1.4	= EN	1.4

Possible reduction acc. to Annex A

	A2.1 reduced geometric deviations due to control $\gamma_c, Red1$	A2.2 (1) measured or reduced geometric data $\gamma_c, Red2$	A2,2 (2) variation coefficient of concrete strength < 10 % $\gamma_c, Red3$	A2.3 concrete strength in the mixing plant determines the diminishing factor $\eta$ ( $\gamma_c, Red * \eta$ )	A2.3 Minimum $\gamma_c$ ( $\gamma_c, Red4$ )
EN	1.4	1.45	1.35	0.85	1.30
NA-D	1.5	1.5	1.5	0.9	1.35
NA-GB	= EN	= EN	= EN	= EN	= EN
NA-A	= EN	= EN	= EN	= EN	= EN
NA-I	1.4	Not allowed	Not allowed	Not allowed	1.4
NA-PL	1.35	Not allowed	Not allowed	Not allowed	1.35

$\epsilon_{c2}$ :

- NA-D: In combination with small eccentricities  $e_d/h < 0.1$ ,  $\epsilon_{c2}$  may be assumed with 2.2 ‰.  
This assumption is implemented, except for annular and rectangular hollow cross sections and polygonal cross sections. With these cross sections, always  $\epsilon_{c2}$  acc. to tab. 9.10 is used in the calculation.

$\epsilon_{cu}$ :

- All: According to 6.1.(5), the compressive strain in the plate centre of structured cross sections is to be limited to  $\epsilon_{c2}$  according to tab. 3.1. This limitation is implemented, except for annular and rectangular hollow cross sections and polygonal cross sections.

$f_{yk}$  characteristic value of the yield strength

$f_{tk}$   $k \cdot f_{yk}$  characteristic value of the tensile strength

$\gamma_s$ : partial safety coefficients for reinforcing steel NDP

	Permanent/transient 2.4.2.4	Accidental 2.4.3.4	Earthquake
EN	1.15	1.0	1.15
NA-D	= EN	= EN	= EN
NA-GB	= EN	= EN	= EN
NA-A	= EN	= EN	= 1.0
NA-I	= EN	= EN	= EN
NA-PL	= EN	= EN	= EN

Possible reduction acc. to Annex A

	A2.1 reduced geometric deviations due to control $\gamma_s, Red1$	A2.2 (1) measured or diminished geometric data $\gamma_c, Red2$
NA-EN	1.10	1.05
NA-D	1.15	1.15
NA-GB	= EN2	= EN2
NA-A	= EN2	= EN2
NA-I	Impossible	Impossible
NA-PL	= EN	= EN

The inclination of the upper branch of the internal action curve of the reinforcing steel is taken into account, unless you have unticked this option in the B2 [configuration](#) section.

Minimum moment:  $M > N \cdot \max(2 \text{ cm}, h/30)$  applies acc. to 6.1 (4)

**NA-D:**

Not required in a second order analysis.

## Design for a given reinforcement ratio

This function is particularly suitable for the design calculation when compressive force with low eccentricity applies. It can also be used universally, however, with multiaxial loading and circular cross sections, for instance. The breaking state is assessed by iterative calculation with a given reinforcement layout (biaxial loading) and/or a given ratio of tensile and compression reinforcement (uniaxial loading).

You can reduce the required steel quantity by selecting a particular reinforcement ratio or layout.

### Minimum reinforcement

Where compression members ( $ed/h < 3.5$ ) are concerned, the software checks automatically whether a design of the minimum reinforcement will become decisive.

For the design types uniaxial design of T-beams, rectangular and layered cross sections, the software checks in addition whether the required minimum reinforcement for components under bending will become decisive.

For the design types biaxial design of rectangular and circular cross sections, the minimum reinforcement is currently not considered.

You can optionally disable the consideration of both minimum reinforcements in the section

→ B2 [design configuration](#).

### EN 1992-1-1

NA-D:	Tables for uniaxial loading in / 46 / ( $f_{ck} \leq 50 \text{ N/mm}^2$ ) Circular and rectangular cross sections with $d_1/h = 0.05 \dots 0.20$
NA-A:	Tables for uniaxial loading in / 48 / ( $f_{ck} \leq 50 \text{ N/mm}^2$ ) Circular and rectangular cross sections with $d_1/h = 0.05 \dots 0.20$
NA-GB:	Tables for uniaxial loading in / 50 / ( $f_{ck} \leq 50$ , $f_{ck} = 90 \text{ N/mm}^2$ ) Circular and rectangular cross sections with $d_1/h = 0.05 \dots 0.20$
NA-I:	Exemplary table for uniaxial loading in /58/ ( $f_{ck}=30 \text{ N/mm}^2$ ) Rectangular cross section with $d_1/h = 0.1$
NA-PL	Exemplary tables for uniaxial loading in /64/ ( $f_{ck} \leq 50 \text{ N/mm}^2$ ) Rectangular cross section

## Dimension-dependent design (kd method)

The method is used for the design of cross sections under uniaxial loading and is the preferable method for bending and longitudinal force with high eccentricity.

$$k_d = \frac{d[\text{cm}]}{\sqrt{\frac{M_s[\text{kNm}]}{b[\text{m}]}}} \text{ is the measure of the effect of the cross section loading.}$$

In the first place, the layout of a tensile reinforcement is assumed. The resisting moment for a strain state is calculated via the balance of the moments in regard to the reinforcement layer. The full utilization of the reinforcement produces the strain state with the maximum moment with the compressive limit strain of the concrete on the pressure side and the yield strain at the level of the reinforcement layer. If the applied internal moment is smaller than the limit moment, the breaking state is determined by iterative balancing of the moments and the axial forces. If the applied internal moment is greater than the limit moment, the strain state described above is assumed. The differential moment is balanced with compression reinforcement.

If compressive strains do not occur, the design is performed according to the lever principle.

In linear elastic calculations of continuous beams, the compression zone height should be limited if no constructive measures are undertaken. Compliance with this criterion is achieved by modifying accordingly the limit steel strain that requires the calculation of compression reinforcement.

### Minimum reinforcement

Where compression members ( $ed/h < 3.5$ ) are concerned, the software checks automatically whether a design of the minimum reinforcement will become decisive.

For the design types uniaxial design of T-beams, rectangular and layered cross sections, the application checks in addition whether the required minimum reinforcement for components under bending will become decisive.

You can optionally disable the consideration of both minimum reinforcements in the section  
→ B2 [design configuration](#).

### Specialities in the analyses on continuous beams without redistribution of the internal forces

The criterion for the calculation of a compressive reinforcement is whether the related compression zone height is exceeded. The compression zone height is calculated in accordance with 5.5 (4) with  $\delta = 1.0$  (no redistribution).

$$\frac{x}{d} = \frac{(\delta - k1)}{k2} \quad \text{or} \quad \frac{x}{d} = \frac{(\delta - k3)}{k4} \quad \text{for } f_{ck} > 50 \text{ N/mm}^2$$

	K1	K2	x/d	K3	K4	x/d (C90)
NA-EN	0.44	$k4 = 1.25 (0.6 + 0.0014 / \epsilon_{cu2})$	0.448	0.54	$k4 = 1.25 (0.6 + 0.0014 / \epsilon_{cu2})$	0.33
NA-D	0.64	0.8	0.45	0.72	0.8	0.35 *a)
NA-GB	0.4	$k4 = (0.6 + 0.0014 / \epsilon_{cu2})$	0.6	0.4	$k4 = (0.6 + 0.0014 / \epsilon_{cu2})$	0.53
NA-A	= EN	= EN	= EN	= EN	= EN	= EN
NA-I	= EN	= EN	= EN	= EN	= EN	= EN
NA-PL	= EN	= EN	= EN	= EN	= EN	= EN

NA-D \*a): applies also to lightweight concrete

## Minimum reinforcement for components exposed to bending

The minimum value of a longitudinal reinforcement exposed to tensile stress in accordance with 9.2.1.1 is a NDP.

	Asmin
EN	$= 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_t \cdot d > 0.0013 \cdot b_t \cdot d$
NA-D	$= \frac{M_{cr}}{(f_{yk} \cdot z) + N} / f_{yk}$ with $M_{cr} = (f_{ctm} + \frac{N}{A_c}) \cdot W_c$ and $z = 0.9 \cdot d$ see /14/
NA-GB	= EN
NA-A	= EN
NA-I	= EN
NA-PL	= EN

## Minimum reinforcement for compression members

In accordance with DIN 992-1-1/NA (NCI to 1.5.2.) compression members are cross sections under compression with a related eccentricity of  $ed/h \leq 3.5$ . in the ultimate limit state. If biaxial loading applies, the criterion must be met in one of the two directions at least.

As,min	Columns	Walls
NDP As,min	Columns (9.5.2(2))	Walls (9.6.2(1))
EN	$= 0.10 \cdot \frac{N_{Ed}}{f_{yd}} > 0.002 \cdot A_c$	$= 0.002 \cdot A_c$
NA-D	$= \frac{0.15 \cdot N_{Ed}}{f_{yd}}$	$= 0.15 \cdot \frac{N_{Ed}}{f_{yd}}$ $0.003 \cdot A_c > A_s > 0.0015 \cdot A_c$
NA-GB	= EN	= EN
NA-A	$= 0.13 \cdot \frac{N_{Ed}}{f_{yd}} > 0.0026 \cdot A_c$	= EN
NA-PL	= EN	= EN

## Lever principle

If the resulting longitudinal tensile force lies in the area of the reinforcement layers, no concrete compression zone is produced. To simplify the calculation, it is assumed that the reinforcement reaches the yield limit on bottom and on top. The size of the reinforcement then simply depends on the reinforcement spacing referenced to the centre of gravity of the cross section and the eccentricity of the resulting force and can be calculated according to the lever principle (DafStb H.220 1.2.8).

See in addition → [Calculation of the effective stiffness.](#)

## Calculation of the effective stiffness

The state of strain in which the external and internal forces are in balance is sought after.

The calculation is based on three non-linear equations with three border strains as unknowns. They are resolved by iteration with the help of the Newton method.

The effective stiffness in combination with bending is consequently determined by the strains. The following equations apply

$$E I_{y,eff} = M_y \cdot H / (\varepsilon_1 - \varepsilon_3) \text{ and}$$

$$E I_{z,eff} = M_z \cdot B / (\varepsilon_1 - \varepsilon_2) .$$

H,B: dimensions of the enclosing rectangle of the cross section

$\varepsilon_1$ : Strain with maximum compression

$\varepsilon_2$ : Strain in the adjacent corner in x-direction

$\varepsilon_3$ : Strain in the adjacent corner in y-direction

*Note concerning polygonal cross sections:*

*With general cross sections, uniaxial loading can also produce curvatures in the direction where the moment is equal to zero.*

*Therefore, you should take the curvatures instead of the effective stiffness into account in deformation calculations.*

### External and internal forces

You can optionally select whether the effective stiffness should be calculated in the serviceability limit state (SLS) or the ultimate limit state (ULS), → see [Design configuration](#).

The resulting internal forces are determined by the internal action curves for concrete and steel.

### EN 1992-1-1, ultimate limit state

Internal action curve of steel      Bilinear internal action curve as per figure 3.8 with the design values  $f_{yd}$  (yield limit) and  $f_{td}(\varepsilon_{ud})$ .

Additional option: "Mean values of material parameters":

$$f_y = f_{yk} \text{ and}$$

$$f_t(\varepsilon_{uk}) = f_y \cdot k(\varepsilon_{uk}, k \text{ as per Annex C})$$

NA-D: Figure 3.8.1, NCI to 5.7

$$f_y = 1,1 \cdot f_{yk} \text{ and}$$

$$f_t(\varepsilon_{uk}) = f_y \cdot k(\varepsilon_{uk}, k \text{ as per Annex C})$$

Internal action curve of concrete

If the stress-strain curve is enabled for the calculation of the internal forces (→ see B2 [configuration](#)), the internal action line of concrete as per figure 3.2 and 5.8.6 (3) applies with  $f_c = f_{cd}$  and  $k = E_{cm} / \gamma_{cE} \cdot \varepsilon_{c1} / f_c$ , ( $E_{cm}$ ,  $\varepsilon_{c1}$  and  $\varepsilon_{c1u}$  as per table 3.1 or table 11.3.1.  $\gamma_{cE}$  is a NDP). If it is not enabled, the parabola rectangle diagram in accordance with fig. 3.3 and the parameters as per table 3.1 or 11.3.1 apply.

	$f_c$	$\gamma_{cE}$
EN	$f_{cd}$	1.2
NA-D	$f_{cm}/\gamma_c$	1,5
NA-GB	=EN	= EN
NA-A	=EN	= EN
NA-I	= EN	= EN
NA-PL	= EN	= EN

Additional option "mean values of material parameters"

NA-D: 5.7 (6) et seq., supplementing NCCI

$$f_c = 0.85 \cdot \alpha_{cc} \cdot f_{ck}$$

$$k = E_{cm} \cdot \varepsilon_{c1} / f_c \text{ (} E_{cm}, \varepsilon_{c1} \text{ and } \varepsilon_{c1u} \text{ as per table 3.1 or table. 11.3.1).}$$

Other NAs as NA-D

### EN 1992-1-1, serviceability limit state

Intern. action curve steel Bilinear stress-strain curve, material coefficients are set to 1.0

Intern. action curve concrete Linear internal action curve with  $E_{cm}$

Internal forces In the serviceability limit state SLS, the internal design forces of the ultimate limit state ULS are divided by a factor defined in the configuration or the internal forces of the quasi-permanent load combination are used → see B2 [configuration](#).

### Creep and shrinkage

If creep and shrinkage are enabled in the →B2 [configuration](#), they are considered in the stiffness calculation as follows:

Creep: If the stress-strain curve of the concrete is non-linear (normally in the ULS), strain is modified in the calculation of the internal forces as per 5.8.6 (4)  
 $\varepsilon = \varepsilon / (1 + \varphi)$  with  $\varphi = \varphi(t_0, \infty)$  as per Annex B

In order to take a diminished creep coefficient  $\varphi_{eff}$  as per 5.8.4. into consideration, the user must enter it manually

→ see B2 [Environmental conditions/creep coefficient](#).

With a linear stress-strain curve, the software reduces the modulus of elasticity of the concrete as per eq. 7.20 with

$$E_{ceff} = E_{cm} / (1 + \varphi) \text{ in the calculation of curvatures in state I.}$$

Shrinkage in state I:

Shrinkage is considered via an additional curvature

$$1/r_S = \gamma_{cs} \cdot E_s/E_{ceff} \cdot S/I \quad (\text{equation 7.21})$$

$\varepsilon_{cs}$ : shrinkage strain as per 3.1.4 and Annex B

S: static moment of the reinforcement relative to the centroid axis (state I) or the neutral axis (state II)

I: moment of inertia of the cross section (state I)

Shrinkage in state II:

According to /24/ p. 18, creep is taken into account via a negative compressive pre-strain of  $\varepsilon_{cs}$  in the calculation of the internal steel forces.

### Tension stiffening

If the corresponding option is activated in the → B2 [configuration](#), tension stiffening or the participation of the concrete between the cracks is considered by modifying the internal action curve of the reinforcing steel (cf. /14/ p. 35). Depending on the relationship between the steel strain under load in state II and the steel strain under internal crack forces, the steel strain is reduced due to tension stiffening acc. to /14/ figure H.8-3 to  $\varepsilon_{sm}$ .

Component stiffness : Only with the cross section types rectangle uniaxial, T-beams and layered cross section.

In accordance with equation 7.18, the distribution coefficient  $\zeta$  provides for a weighting between

the curvatures in state II  $1/r_{II} = (\varepsilon_2 - \varepsilon_1) / h$  and

the curvatures in state I  $1/r_I = M / (I_i \cdot E_{ceff}) + 1/r_S$

to an average curvature  $1/r_m = 1/r_{II} \cdot \zeta + (1-\zeta) \cdot 1/r_I$

$$\zeta = 1 - \beta \cdot (\sigma_s/\sigma_{sr})^2 \quad \text{equation 7.19}$$

$\sigma_{sr}$ : steel strain in state II exposed to internal crack forces calculated with  $f_{ctk0.05}$  (default) or  $f_{ctm}$  (option),  
→ see B2 [design configuration](#).

$\sigma_s$ : steel strain in state II under the load for which the stiffness is calculated (default) or in the infrequent load combination (option),  
→ see B2 [design configuration](#)

Short-term loading:  $\beta = 1.0$  (ULS)

Long-term loading:  $\beta = 0.5$  (SLS)

$$E_{leff} = M_y / (1/r_m)$$

Cross-sectional stiffness: The effective stiffness is determined by the curvatures in state II using the factor  $k\zeta = (\varepsilon_{sm} - \varepsilon_{c2}) / (\varepsilon_{s2} - \varepsilon_{c2})$  to obtain  
 $E_{leff} = M / (k\zeta \cdot 1/r_{II})$  (cf. /22/ p. 303)

## Shear design

### Shear force

The analysis of the shear resistance is based on a truss model with compressive concrete struts and steel ties (stirrups). The minimum stirrup requirements result from the flattest possible strut inclination.

A flatter inclination reduces the bearing capacity of the struts, however, and increases in addition the forces in the tension chord. The result is an increased offset dimension.

### Shear design for vertical shear reinforcement (stirrups):

**VEd** design value of the shear force (ULS)

**VRd,c** The shear resistance without reinforcement for the cracked state results from equation 6.2 or 11.6.2 for lightweight concrete

$$VR_{d,c} = CR_{dc} \cdot \eta_1 \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \cdot b_w \cdot d \geq VR_{dc} \text{ (eq. 6.2b)}$$

CR<sub>dc</sub>: calibration factor acc. to 6.2.2: (1) (NDP)

K<sub>1</sub>: empirical strain coefficient

NDP	k <sub>1</sub> :	CR <sub>dc</sub>
EN	0.15	0.18/γ <sub>c</sub> standard concrete 0.15/γ <sub>c</sub> lightweight concrete
NA-D	0.12	0.15/γ <sub>c</sub>
NA-GB	0.15,	0.18/γ <sub>c</sub> , > C50 test or as C50
NA-A	= EN	= EN
NA-I	= EN	= EN
NA-PL	= EN	= EN

η<sub>1</sub> correction factor for lightweight concrete

$$k = 1 + \sqrt{200/d} \leq 2 \quad [d \text{ in mm}]$$

scaling factor, decreases when the effective height increases

$$\rho_l = A_{sl} / (b_w \cdot d) < 0.02$$

tensile reinforcement A<sub>sl</sub> that goes beyond the considered cross section with l<sub>bd</sub>+d

$$\sigma_{cp} = N_{Ed} / A_c < 0.2 \cdot f_{cd}$$

stress (negative compression)

b<sub>w</sub>: lowest cross section width within the effective height

Equation 6.2.b

$$VR_{d,c} > (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$$

NDP	v <sub>min</sub> standard concrete	v <sub>l,min</sub> lightweight concrete
EN	$0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$	$0.028 \cdot k^{3/2} \cdot f_{ck}^{1/2}$
NA-D	$0.0520/\gamma_c \cdot k^{3/2} \cdot f_{ck}^{1/2}$ (d < 600) $0.0375/\gamma_c \cdot k^{3/2} \cdot f_{ck}^{1/2}$ (d > 800)	0
NA-GB	= EN	$0.028 \cdot k^{3/2} \cdot f_{ck}^{1/2}$
NA-A	= EN	
NA-I	= EN	$0.030 \cdot k^{3/2} \cdot f_{ck}^{1/2}$
NA-PL	= EN	= EN

NA-GB: > C50 with f<sub>ck</sub> = 50 N/mm<sup>2</sup> or additional option "no reduction"

Optionally, the user can perform a calculation in the uncracked state as per equation 6.4 (see [B2 configuration](#)), if the concrete border strain is smaller than f<sub>ctk</sub> 0.05/γ<sub>c</sub> (NA-D: f<sub>ctd</sub>).

NA-D: does not apply to pre-stressed element ceilings

Alternative: applies to single-span systems of pre-stressed concrete

$$V_{Rd,c} = \frac{I \cdot b_w}{S} \sqrt{(f_{ctd})^2 + \alpha_l \cdot \sigma_{cp} \cdot f_{ctd}}$$

I: moment of inertia

S: static moment in the decisive section

b<sub>w</sub>: width in the decisive section

σ<sub>cp</sub>: longitudinal stress in the decisive section

α<sub>l</sub>: coefficient for pre-tensioning in the area of the transmission length, otherwise always 1.0

f<sub>ctd</sub>: arithmetical value of the tensile strength of the concrete

$$f_{ctd} = \alpha_{ct} \cdot f_{ctk} \cdot 0.05 / \gamma_c$$

γ<sub>c</sub>: partial safety factor (see [Bases of design](#))

f<sub>ctk</sub> 0.05: lower characteristic value of the tensile strength of the concrete

NDP	α <sub>ct</sub> standard concrete as per 3.1.6	α <sub>ct</sub> standard concrete as per 11.3.5
EN	1.0	0.85
NA-D	0.85	0.85
NA-GB	= EN	= EN
NA-A	= EN	= EN
NA-I	= EN	= EN
NA-PL	= EN	= EN

When using equation 6.4 make sure that the decisive section is not in the centre of gravity of the cross section. It should be determined by iteration if the cross section width varies or the longitudinal tension is inconstant. This means that VR<sub>d,c</sub> also depends from the entered longitudinal force (minimum is decisive) and the entered bending moment (maximum is decisive).

### Components with required shear reinforcement

**Cot  $\Theta$**  The goal of the design is to minimize shear reinforcement, i.e. the flattest possible strut inclination angle (max Cot  $\Theta$ ) is sought after, at which the bearing capacity of the strut is still ensured.

If loading by torsion applies simultaneously, this bearing capacity can become decisive for the strut inclination angle to be selected.

NDP	Max Cot $\Theta$	Min Cot $\Theta$	Comment
EN	2.5	1.0	Determination of $\Theta$ based on $VR_{d,max}$ criterion
NA-D	3.0 standard concrete 2.0 lightweight concrete	0.58	Take additional crack fraction criterion into account
NA-GB	= EN2 1.25 with external tension	= EN2	= EN
NA-A	1.6 in general 2.5 with overpressure on cross section	= EN2	= EN
NA-I	= EN	= EN	= EN
NA-PL	2.0	= EN	= EN

#### NA-D:

$$\text{Cot } \Theta \leq (1.2 - 1.4 \cdot \sigma_{cd}/f_{cd}) / (1 - VR_{d,cc}/VE_d) \quad \text{eq. 6.7aDE}$$

**VR<sub>d,cc</sub>:** Crack friction force

$$VR_{d,cc} = \beta_{ct} \cdot 0.1 \cdot f_{ck}^{1/3} \cdot (1 - 1.2 \cdot \sigma_{cd}/f_{cd}) \cdot b_w \cdot z \quad \text{eq. 6.7.bDE}$$

You can optionally set the strut inclination angle by default

(→ B2 [design options](#)) to analyze additional sections with the strut inclination angle relevant at the decisive cross section, for instance. This angle must not be flatter than the required one.

**z** lever arm of the assumed framework model according to the bending design (if unknown, assumption of  $0.9 \cdot d$ , or of  $0.55 \cdot d$  with circular cross sections).

**NA-D:** limitation  $z < d - 2 \cdot c_{v,l}$  (here  $c_{v,l}$  = nomc of the longitudinal reinforcement in the compression zone, acc. to /26/, a limitation of  $z < d - c_{v,l} - 3\text{cm}$  applies to  $c_{v,l} > 3\text{cm}$ ).

You can also set a user-defined lever arm by default

(→ B2 [design results](#)).

**aswV** calculated shear reinforcement acc. to equation 6.8

The selection of the strut inclination angle also in line with the criterion for compliance with  $VR_{dmax}$  proves equation 6.12.

The software checks whether a minimum shear reinforcement acc. to 9.2.2 (5) for beams or 9.3.1.4 (NAD\_D) for plates will become decisive. The reinforcement is calculated for an average web width (with circular cross sections  $b_w S = A_c/D_a$ ).

With circular cross sections, an efficiency factor for round stirrups is calculated in accordance with /31/ that increases the required shear reinforcement. This factor takes into account that the applying shear force is normally not parallel to the resisting force of the stirrup. Depending on the considered section, the resisting force applies at a different angle to the perpendicular.

Min  $asw/s = \rho \cdot bw \cdot \sin \alpha$

	$\rho$ (beams) as per 9.2.2:	$\rho$ (plates) as per 9.3.2:	Comment
EN	$0.08 \cdot \sqrt{f_{ck}/f_{yk}}$	0	
NA-D	$0.16 \cdot f_{ctm}/f_{yk}$	0 if $V_{Ed} < VR_{dc}$ Otherwise $0.6 \cdot \rho$	Junction area $4 < b/h < 5$ : Interpolation between 0 and the simple value ( $V_{Ed} < VR_{dc}$ ) or between 0.6 and the simple value ( $V_{Ed} > VR_{dc}$ )
NA-GB	= EN	= EN	
NA-A	$0.15 \cdot f_{ctm}/f_{yk}$	= EN	
NA-I	= EN	= EN	Draft NA
NA-PL	= EN	= EN	

**$VR_{d,max}$**  The bearing capacity of the struts results acc. to 6.9 or equivalent and depends only on  $\cot \Theta$ . The following equation applies:

$$VR_{d,max} = bw \cdot z \cdot \alpha_{cw} \cdot v_1 \cdot f_{cd} \cdot \cot \Theta / (1 + \cot^2 \Theta)$$

NDP	$v_1$ acc. to 6.2.3	Comment
EN	$v_1 = 0.6 \cdot (1 - f_{ck}/250)$ $v_1 = 0.5 \cdot (1 - f_{ck}/250)$	equation 6.6N equation 11.6.6N lightweight concrete
NA-D	$v_1 = 0.75$ * $(1.1 - f_{ck}/500)$ * $\eta_1$	Standard concrete > C50 Lightweight concrete
NA-A	= EN	
NA-GB	$v_1 = 0.6 \cdot (1 - f_{ck}/250)$ $v_1 = 0.5 \cdot (1 - f_{ck}/250)$	equation 6.6N equation 11.6.6N lightweight concrete
NA-I	$v_1 = 0.5$ [1] $v_1 = 0.5 \cdot \eta_1 (1 - f_{ck}/250)$ [4]	Standard concrete Lightweight concrete
NA-PL	= EN	

**All NAs:** the increase by including only 80 % of the stirrup bearing capacity in acc. with equation 6.10a and 6.10b is not considered.

For reinforced concrete:  $\alpha_{cw} = 1.0$

**NA-GB:** > C50 with  $f_{ck} = 50 \text{ N/mm}^2$  or additional option "no reduction"

(see [B2 configuration](#))

PD 6687:2006 chapter 2.3 allows the calculation of  $f_{cd}$  with  $\alpha_{cc} = 1.0$ .

(Option "increased  $f_{cd}$  as per PD 6687:2006" see [B2 configuration](#))

The maximum of  $VR_{d,max}$  results for a strut inclination angle of  $45^\circ$ .

If  $VR_{d,max}$  is smaller than the design value of the shear force, you should increase the cross section or the concrete class.

**bw** The width  $b_w$  corresponds to the web width  $b_0$  for T-beams and to the lowest width in the cross section for layered cross sections. Where circular cross sections are concerned,  $b_w$  corresponds to the lowest width between the resultant compression force and the resultant tension force. If the position of the resultant force is unknown (moment and axial force are equal to zero) a safe distance of the resultant compression force of  $D_a/40$  is assumed in the calculation.

**sl,max** maximum stirrup spacing as per 9.2.2 (6)

	sl,max (NDP acc. to 9.2.2 (6))
EN	$0.75 \cdot d \cdot (1 + \cot \alpha)$
NA-D	distinguished according to shear force utilization with a $V_{Rdmax}$ ( $\Theta = 40^\circ$ )
NA-GB	= EN2
NA-A	$0.75 \cdot d \cdot (1 + \cot \alpha) \leq 250 \text{ mm}$
NA-I	= EN
NA-PL	= EN

**NA-D:**

$V_{Ed} < 0.3 \cdot V_{Rdmax}$        $s_{Max} = 0.7 \cdot h$  beam:  $< 30 \text{ cm}$  ( $> C50/60$ :  $< 20 \text{ cm}$ )

$V_{Ed} < 0.6 \cdot V_{Rdmax}$        $s_{Max} = 0.5 \cdot h$  beam:  $< 30 \text{ cm}$  ( $> C50/60$ :  $< 20 \text{ cm}$ )

$V_{Ed} > 0.6 \cdot V_{Rdmax}$        $s_{Max} = 0.25 \cdot h$  beam:  $< 20 \text{ cm}$

$V_{Rdmax}$  may be assumed with  $\theta = 40$  degrees according to /14/ p. 212

### Biaxial shear force for rectangular cross sections

In accordance with the method described in reference /39/, the verification is reduced to the uniaxial scenario with the help of adjusting factors for the load-bearing capacity of the struts and the stirrups.

**Boundary case 1** is uniaxial loading with  $\alpha_v = 0$ , **boundary case 2** is biaxial loading with an accurately diagonal load application of the resultant, i.e.  $\alpha_v = 1$ .

In accordance with reference /39/, the force in the stirrup for case 2 is as follows:

$$2 \cdot V_z = 2 \cdot \frac{V_{Ed}}{\sqrt{\left(\frac{b}{h}\right)^2 + 1}}, \text{ which means it is } \frac{2}{\sqrt{\left(\frac{b}{h}\right)^2 + 1}} \text{ times greater than in case 1.}$$

The highest loading on the compressive concrete strut results in case 2 for the load transfer point from the strut to the tension chord, where the width  $b_{eff}$  is reduced to  $0.6 \cdot b$  according to the conservative estimation prescribed in reference /39/. When assuming the same lever arm in both cases, the compressive strut loading resulting in case 2 is  $b/b_{eff}$  times higher than in case 1.

Between these two cases, interpolation is performed in accordance with the existing inclination  $\alpha_v$  with the help of the following relations:

$$V_{Ed}: \quad \text{resulting shear force } \sqrt{V_{Edy}^2 + V_{Edz}^2}$$

$$\alpha_v: \quad \text{related shear force inclination } \frac{|V_{Edy}| \cdot h}{|V_{Edz}| \cdot b}$$

$h$ : side length in the  $z$ -direction

$b$ : side length in the  $y$ -direction

if  $0 \leq \alpha_v \leq 1$ ,

then bearing strength verification with  $b_w = b$ ,

otherwise  $\alpha_v = 1/\alpha_v$  and bearing strength verification with  $b_w = h$ ,

$$V_{Rd,sy} = V_{Ed} = \frac{A_{sw}}{s_w} \cdot f_{yd} \cdot z \cdot \cot\theta \cdot \frac{1}{k_{asw}}$$

### Interpolation factor for shear reinforcement

$$k_{asw} = 1 + \left( \frac{2}{\sqrt{\left(\frac{b}{h}\right)^2 + 1}} - 1 \right) \cdot \alpha_v^{1/2}$$

$$V_{Rd,max} = b \cdot z \cdot \alpha_c \cdot \frac{f_{cd}}{\cot\theta + 1/\cot\theta} \cdot k_{vmax}$$

**Interpolation factor for compressive strut resistance**

$$k_{vmax} = \frac{1}{1 + \left( \frac{b}{b \cdot 0,6} - 1 \right) \cdot \alpha_v^{1/2}}$$

z lever arm of the inner forces, i.e. the distance between the tension resultant and the compression resultant, as resulting from the bending design.

If the lever arm is unknown, interpolation is performed between  $z = 0.9 \cdot (h-d1)$  for  $\alpha_v = 0$  and  $z = 0.9 \cdot (h-d1+b-1)/2$  for  $\alpha_v = 1.0$  in relation to the existing  $\alpha_v$ .

NA\_D:  $z < d - 2 \cdot nomc$

This limitation shall ensure that the distance of the compression resultant to the compressive edge is not smaller than  $2 \cdot nomc$ .

Consequently, d refers to the distance of the tension resultant for the compressive edge in the direction of the lever arm.

$V_{Rdc}$  is calculated by approximation with  $b_w = 0.6 \cdot b_w$  (case 1) and  $d = z$ .

**Cast-in-place complement**

For cross sections with cast-in-place complement, the bearing capacity of the cast-in-place joint is to be verified  $vEd_i < vRd_i$  equation 6.23

**vEd<sub>i</sub>** shear force to be transmitted per length unit in the joint

$$vEd_i = \beta \cdot VEd / (z \cdot b_i) \quad \text{equation 6.24}$$

**VEd:** design value of the shear force

z: lever arm of the internal forces,  
see shear resistance verification

NA\_D: if  $V_{Rd,c} > VEd$ , the lever arm limitation with  $c_v$  can be dispensed with.

$\beta$ : ratio of axial force in the cast-in-place concrete to total compression force (assumption 1.0)

**vRd<sub>i</sub>** design value of the shear force resistance of the joint

$$vRd_i = c \cdot f_{ctd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} \cdot (\mu \cdot \sin \alpha + \cos \alpha) < 0.5 \cdot v \cdot f_{cd}$$

(equation 6.25, lightweight concrete with  $d_{ctd} = f_{lctd}$  and  $v = v_l$  and  $f_{cd} = f_{lcd}$ )

**$\sigma_n$**  axial stress perpendicular to the joint with  $\sigma_{ND} = nEd/b_i < 0.6 \cdot f_{cd}$

nEd: design value (compression: lower, tension: upper) of the axial force perpendicular to the joint per length unit, negative compression.

b<sub>i</sub>: effective joint width, reduced total width due to prefabricated formwork, if applicable.

**c** roughness coefficient according to surface quality

Very smooth	Smooth	Rough	Interlocked
0.1	0.20	0.40	0.50

**$\mu$**  friction coefficient according to surface quality as per table 13

Very smooth	Smooth	Rough	Interlocked
0.5	0.6	0.7	0.9

**v** strength reducing coefficient as per 6.2.2 (6)

v	Very smooth	Smooth	Rough	Interlocked
EN				
Standard concrete	$0.6 \cdot (1-f_{ck}/250)$	$0.6 \cdot (1-f_{ck}/250)$	$0.6 \cdot (1-f_{ck}/250)$	$0.6 \cdot (1-f_{ck}/250)$
Lightweight concrete	$0.5 \cdot (1-f_{ck}/250)$	$0.5 \cdot (1-f_{ck}/250)$	$0.5 \cdot (1-f_{ck}/250)$	$0.5 \cdot (1-f_{ck}/250)$
NA-D (NCCI)				
Standard concrete	0.0	0.2	0.5	0.7
> C50	0.0	* $(1.1-f_{ck}/500)$	* $(1.1-f_{ck}/500)$	* $(1.1-f_{ck}/500)$
Lightweight concrete	* $\eta_1$	* $\eta_1$	* $\eta_1$	* $\eta_1$

**NA-D:**

$$vR_{di} = c \cdot f_{ctd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} \cdot (1.2 \cdot \mu \cdot \sin \alpha + \cos \alpha) < 0.5 \cdot v \cdot f_{cd}$$

(eq. 6.25 + NCI or eq. 11.6.25 for lightweight concrete, with  $f_{ctd} = f_{lctd}$  and  $v = v_l$  and  $f_{cd} = f_{lcd}$ )

very smooth with  $c = 0$

**$\rho$**  shear reinforcement ratio of the joint

$$\rho = A_{sw} / A_i = a_{sw} / b_i$$

**asw** required stirrup reinforcement crossing the joint, hence  $vR_{di} = vE_{di}$

$$vR_{di0} = c \cdot f_{ctd} + \mu \cdot \sigma_n \quad \text{bearing capacity without joint reinforcement}$$

$$a_{sw} = b_i \cdot (vE_{di} - vR_{di0}) / (f_{yd} \cdot k \cdot \mu \cdot \sin \alpha + \cos \alpha)$$

NA-A:

$$a_{sw} > \text{Min} = \rho_{\min} \cdot b$$

$$\text{Plates: } \rho_{\min} = 0.12 \cdot f_{ctm} / f_{yk} > 0.0005$$

$$\text{Beams: } \rho_{\min} = 0.20 \cdot f_{ctm} / f_{yk} > 0.001$$

The verification of the anchorage required by the National Annex is not implemented currently. A successful result is presumed, however, because  $a_{sw}$  is calculated with  $f_{yd}$  without reduction.

**Torsion**

Torsion design is done with the help of an equivalent hollow cross section. With structured cross sections, only the web cross section is used in the approach by approximation.

**tef,i:** effective wall thickness

$$tef,i = A / U$$

<  $2 \cdot d_1$  double spacing of reinforcement

<  $b_a$  real wall thickness with hollow cross sections

The requirement to verify explicitly torsional resistance instead of the minimum reinforcement results from the interaction equation 6.31 that is different in NA-D.

**NA-A, NA-GB:**

$$T_{Ed}/TR_{dc} + V_{Ed}/VR_{d,c} < 1 \quad \text{equation 6.31}$$

**T<sub>Ed</sub>:** design value of the torsional moment

**TR<sub>dc</sub>:** resisting torsion moment only depending on the tensile strength of the concrete

$$TR_{dc} = f_{ctd} \cdot t \cdot 2 \cdot A_k \quad \text{as per /55/ p. 6-13}$$

W<sub>t</sub>: section modulus as per /46/ p. 309

**NA-D:**

$$T_{Ed} < V_{Ed} \cdot b_w / 4.5 \quad \text{equation 6.31aDE}$$

$$V_{Ed} \cdot (1 + (4.5 \cdot T_{Ed}) / (V_{Ed} \cdot b_w)) \leq VR_{dct} \quad \text{equation 6.31bDE}$$

**Cot Θ**

The goal of the design is to minimize shear reinforcement, i.e. the flattest possible strut inclination angle (max Cot Θ) is sought after, at which the bearing capacity of the strut is still ensured.

This calculation does not automatically produce the reinforcement minimum because the portion of the longitudinal torsion reinforcement increases considerably with flatter struts.

If shear loading applies simultaneously, the interaction of shear force and torsion might become decisive for the design.

To simplify the calculation, you can base the torsion analysis exclusively on the assumption  $\text{Cot } \Theta = 1.0$  (45 degrees) (see [Design configuration](#)).

**NA-D:**

Calculation of the strut inclination angle acc. to /51/, p. 173 ff

$$\text{Cot } \Theta \leq (1.2 - 1.4 \cdot \sigma_{cd}/f_{cd}) / (1 - VR_{d,cc}/V_{Ed, T+V}) \quad \text{acc. to equation 6.7.aDE}$$

V<sub>Ed, T+V</sub>: resultant loading

$$V_{Ed, T+V} = V_{Ed, T} + V_{Ed, V} \cdot t_{eff, I} / b_w$$

V<sub>Ed, V</sub>: loading by shear force

V<sub>Ed, T</sub>: loading by torsion

$$V_{Ed, T} = T_{Ed} \cdot z_i / (2 \cdot A)$$

VR<sub>d, cc</sub>: crack friction force acc. to eq. 6.7.bDE

$$VR_{d, cc} = \beta_{ct} \cdot 0.1 \cdot f_{ck}^{1/3} \cdot (1 - 1.2 \cdot \sigma_{cd}/f_{cd}) \cdot t_{ef, i} \cdot z$$

**VRd,max** design value of the resisting torsional moment acc. to equation 6.30 or equivalent depending only on  $\cot \theta$ . The following equation applies:

$$Trd,max = 2 \cdot v \cdot \alpha_{cw} \cdot f_{cd} \cdot A_k \cdot t_{ef,l} \cdot \cot \theta (1 + \cot^2 \theta)$$

Ak: area enclosed by the wall centre lines

NDP	v (6.2.2. (6))	Comment
EN	$v = 0.6 \cdot (1 - f_{ck}/250)$ $v = 0.5 \cdot \eta_1 \cdot (1 - f_{ck}/250)$	Analogously to shear force Standard concrete Lightweight concrete
NA-D (NCCI)	$v = 0.525$ $* (1.1 - f_{ck}/500)$ $* \eta_1$	Reduced in comparison to shear force Standard concrete > C50 Lightweight concrete
NA-A	= EN	
NA-GB	= EN	
NA-D (NCCI)	$v = 0.5$ $v = 0.5 \cdot \eta_1 \cdot (1 - f_{ck}/250)$ [4] p. 63	Analogously to shear force Standard concrete Lightweight concrete
NA-PL	= EN	

$\alpha_{cw}$ : coefficient analogous to VRd,max

The maximum for TRd,max results for a strut inclination angle of 45 degrees. If TRd,max is smaller than the design value of the torsional moment, you should increase the cross section or select a higher concrete class.

**aswT** the required stirrup reinforcement due to torsion results from

$$aswT^* = T_{Ed} / (2 \cdot A_k \cdot f_{yd} \cdot \cot \theta) \quad /46/ \text{ p. 283}$$

Because only one leg of torsion stirrups may be considered in the calculation,  $aswT = 2 \cdot aswT^*$  applies.

The minimum shear reinforcement becomes decisive if  $aswV + aswT < asw_{Min}$  is true.

**AsL** additional longitudinal reinforcement due to torsion

$$AsL = T_{Ed} \cdot \cot \theta \cdot U_k / (2 \cdot A_k \cdot f_{yd}) \quad \text{eq. 6.28}$$

Uk: circumference of area Ak

With combined shear force and torsional loading, the following interaction condition must be complied with:

$$T_{Ed} / TR_{d,max} + V_{Ed} / VR_{d,max} < 1 \quad \text{equation 6.29}$$

**NA-D and NA-A:**

For compact cross section applies

$$(T_{Ed} / TR_{d,max})^2 + (V_{Ed} / VR_{d,max})^2 < 1 \quad \text{equation 6.29aDE}$$

The stirrup cross section results from  $asw(V+T) = aswV + aswT$ .

**Shear design for prefabricated floors with lattice girders:**

The verification for DIN EN 1992-1-1/NA can be performed on the bases of manufacturer-specific approvals (e.g. ref. /67/.../72/).

Lattice girders consist of a compression chord, a tension chord and struts.

The struts can either have the shape of isosceles triangles

(inclination angle of  $45^\circ \leq \alpha < 90^\circ$  e.g. ref. /67/, /69/, /71/, referred to as "isosceles triangle" in the following structural system) or consist of a vertical post and a diagonal strut

(inclination angle of  $45^\circ \leq \alpha < 90^\circ$  e.g. ref. /68/, /70/, /72/., referred to as "post/diagonal strut" in the following structural system).

The following limitations apply:

- Permissible only for plates ( $w/h > 5$  or option „Like plate“)
- Minimum thickness of 4 cm
- Concrete grades  $< C50/60$  or  $< LC50/55$  with a raw density class of D1.2
- "Isosceles triangle" system only permissible for mainly steady live loads

Design for shear force resistance:

**VRdc** In derogation of the design standard, longitudinal compression stress must not be taken into account.

**Cot  $\Theta$**  in derogation of the design standard, the lower limit is  $\text{Cot } \Theta \geq 1.0$ .

in derogation of the design standard, longitudinal compression stress must not be taken into account.

**aswQ** the required shear reinforcement is calculated using eq. 6.13 in accordance with the inclination angle  $\alpha$  of the struts. For the system post/diagonal strut, it is assumed that the diagonal strut ( $\alpha=\alpha_1$ ) and the post ( $\alpha=90$  degrees) bear 50 % of the load each.

If the struts are made of smooth reinforcing steel B 500 A+G, a  $f_{yd}$ -value of merely  $f_{yd} = 365 \text{ N/mm}^2$  may be taken into account.

**VRd,max** is calculated using eq. 6.14 in accordance with the inclination angle  $\alpha$  of the struts. In derogation of the relevant standard, the following applies in accordance with eq. 6.14:  
 $VR_{d,max,GT} = 1/3 \cdot VR_{d,max}$ .

For the post/diagonal strut system, the verification is based on an interaction equation  $\Sigma (VR_{dsy, \alpha_i} / VR_{dmax, \alpha_i}) \leq 1,0$  due to the different inclinations of the struts. (See ref. /66/ eq. H.6-7)

$VR_{dsy, \alpha_i}$ : bearing capacity portion of the strut with the angle  $\alpha_i$

$VR_{dmax, \alpha_i}$ : bearing capacity of the compressive strut with assumption of a strut inclination angle  $\alpha_i$

If the verification is not successful, the cross section or the concrete class should be increased.

**sMax** maximum distance of the diagonal strut in the supporting direction as per ref. /67/ to /72/

$$s_{max} = (\cot \theta + \cot \alpha) \cdot z \leq 20 \text{ cm}$$

Shear force transmission in the joint:

In derogation of the verification method described in chapter 6.2.5, the limitation of  $vR_{di,max}$  in accordance with the tables 4a (standard concrete) or 4b (lightweight concrete) applies in addition.

If the verification of the shear force resistance reveals that  $VEd < VR_{dc}$ , the lever arm limit  $z < \max.(d - 2 \cdot c_vl, d - c_vl - 3 \text{ cm})$  is not taken into account in the calculation of  $vEd$ .

(See ref. /66/ concerning 6.2.5 (1))

## Serviceability verifications

### Crack width verification in accordance with EN 1992-1-1

Based on the **crack formula equation 7.8**  $w_k = s_{r,max} \cdot (\epsilon_{sm} - \epsilon_{cm})$

the maximum limit diameter still in compliance with the permissible crack width is calculated for an external loading that depends on the decisive combination of actions and a pre-selected reinforcement.

#### Decisive combinations of actions and permissible crack width as per table 7.1 (NDP)

The considered NAs all require the verification of a permissible crack width of 0.3 mm for reinforced concrete components of exposure class XC2 and higher.

The verification for XC1 is based on a crack width of 0.4 mm for aesthetical reasons (exception GB: 0.3 mm)

Under normal conditions, the quasi-permanent load combination (Qk) is the decisive one.

Considerably different requirements apply in Italy and the Netherlands.

Requirements referring to reinforced concrete components as per table 7.1.

	X0, XC1	XC2/XC4	XS1-3, XD1-3	Comment
EN	0.4 mm + Qk	0.3 mm + Qk	0.3 mm + Qk	Tab. 7.1N
NA-D	= EN	= EN	= EN	Tab. 7.1DE
NA-GB	0.3 mm + Qk	= EN	= EN	
NA-A	= EN	= EN	= EN	
NA-I	AO 0.3 mm + Qk 0.4 mm + Hk	AA 0.2 mm + Qk 0.3 mm + Hk	AM 0.2 mm + Qk 0.2 mm + Hk	A0,A0,AA,AM as per NTC tab. 4.1. III
NA-PL	= EN	= EN	= EN	

Due to the fact that the tensioning steel is highly susceptible to corrosion, pre-stressed concrete components have to comply with higher requirements in regard to the load combinations (infrequent (Sk), frequent (Hk)) to be verified and the permissible crack width. In some cases, a verification of decompression (dec.) might be required.

The regulations differ in the various National Annexes.

Bonded pre-stressed concrete:

	X0, XC1	XC2/XC4	XS1-3, XD1-3	
EN	0.2 + Hk	0.2+ Hk Dec. Qk	Dec. Hk	Tab. 7.1N
NA-D	= EN	= EN	Bonded post-tensioned concrete: 0.2+ Hk and dec. Qk Bonded pre-tensioned concrete 0.2 + Sk and dec. Hk	Tab. 7.1DE
NA-GB	= EN	= EN	= EN	
NA-A	= EN	= EN	Bonded post-tensioned concrete: 0.2+ Hk and dec. Qk Bonded pre-tensioned concrete 0,2 + Sk and dec. Hk	
NA-I	AO 0.3 mm + Qk 0.4 mm + Hk	AA 0.2 + Hk dec.+ Qk	AM dec. + Qk Sig <sub>t</sub> + Sk	A0,AO,AA,AM as per NTC tab. 4.1. III
NA-PL	= EN	= EN	= EN	

The crack width results from the maximum crack spacing  $s_{rmax}$  and the average strain difference  $\epsilon_{sm} - \epsilon_{cm}$  of concrete and steel.

$\epsilon_{sm} - \epsilon_{cm}$ : average strain difference between steel and concrete (equation 7.9)

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

$k_t$ :                    0.6 short-term action (not considered in the software)  
                             0.4 long-term action

$\sigma_s$ :                    steel strain in state II  
                             calculation with  $E_{ceff} = E_{cm} / (1 + \varphi (t = \infty))$

$\alpha_e$                     =                     $E_s / E_{ceff}$

$\rho_{eff}$ :                    reinforcement ratio in the effective tension zone

$$\rho_{eff} = (A_s + A_p * \xi^2) / A_{ceff}$$

$A_s$ :                    reinforcing steel area included in  $A_{ceff}$

$A_p$ :                    tensioning steel area included in  $A_{ceff}$

$\xi$ :                      factor for the bond characteristics of tensioning steel

- $A_{ceff}$  : area of the effective tension zone  
 $A_{ceff} = h_{eff} \cdot b_{eff}$
- $h_{eff}$   $2.5 \cdot D1 < (h-X0II)/2$   
 X0II: compression zone height in state II:  
 if no reinforcement with spacing  $< h_{eff}$   
 was defined,  $h_{eff} = (h-X0I)/2$  applies
- $b_{eff}$  effective tension zone width for T-beams  
 NA-D:  
 as per /5/ p. 191 in accordance with the permissible relocation width of the tensile reinforcement  
 $b_{eff} \leq \sum (0.5 \cdot b_{eff,i}(Z.l)) + bw \leq bf$  (NCI zu 9.2.1.2 (2))  
 Input: see B2 dialog for [Control of the crack width verification](#)

**Sr, max: maximum crack spacing:**

$$s_{r,max} = k_3 \cdot c + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi}{\rho_{p,eff}}$$

- $k_1$ : coefficient reinforcement bond quality  
 0.8 good bond quality  
 1.6 poor bond quality
- $k_2$ : coefficient of strain distribution  
 Bending: 0.5  
 Tension 1.0  
 Bending + tension  $(\varepsilon_1 + \varepsilon_2) / (2 \cdot \varepsilon_1)$
- $c$ : concrete cover on longitudinal reinforcement
- $\phi$ : average diameter of the tensile reinforcement

NDP	$k_3$	$k_4$
EN	3.4	0.425
NA-D	0	$1/(3.6 \cdot k_1 \cdot k_2) < \phi \cdot \sigma_s / (3.6 \cdot f_{ct,eff})$
NA-GB	= EN	= EN
NA-A	0	$1/(3.6 \cdot k_1 \cdot k_2) < \phi \cdot \sigma_s / (3.6 \cdot f_{ct,eff})$
NA-I	= EN	= EN
NA-PL	= EN	= EN

NA\_D: For lattice girders with approval by the construction authorities, ref. /67/ .../72/ with smooth reinforcing steel in the chord, reduced bond stress can be taken into account.  
 In accordance with the bond stress for smooth bars specified by e.g. DIN 1045 /78 a factor of 1/0.388, which is on the safe side, results for the crack width.  
 This factor is also suitable for the calculation of the limit diameters specified in the tables of the approvals.

The limit diameter  $\phi$  is obtained by rearranging the crack equation.

More favourable (larger) limit diameters than specified in table 7.2 may result because the simplifications the table is based on are dispensed with.

If the resultant limit diameter cannot be realized, you should increase the selected reinforcement.

For circular cross sections,  $\rho_{\text{eff}} = A_s/A_{c,\text{eff}}$  is calculated for a circular ring with a thickness of  $h_{\text{eff}}$  because an evenly distributed reinforcement is assumed in accordance with reference /30/.

The expression  $A_{c,\text{eff}} = \pi (D \cdot h_{\text{eff}} - h_{\text{eff}}^2)$  allows a more accurate determination.

The condition  $A_{c,\text{eff}} \leq A_c$  applies to circular ring cross sections in addition.

The results comply well with reference /30/ if the specified condition of  $n = 10$  is satisfied by taking low creep factors into account. The results for  $t = \infty$  are less favourable, however, because the creep factors are higher then.

### Minimum reinforcement due to indirect action

The software application calculates a minimum reinforcement acc. to 7.3.2 for imposed bending on top and bottom if the corresponding option was enabled in the [Control of the crack width verification](#) dialog.

The minimum reinforcement for T-beams is calculated separately for the web and the flange, whereby the rectangle over the total cross section height is considered as the web and the remaining parts of the plate as the flange. You can take different bar diameters for flange and web into account.

$$A_{s,\text{min}} \cdot \sigma_s = k_c \cdot k \cdot f_{\text{ct,eff}} \cdot A_{\text{ct}} \quad (\text{equation 7.1})$$

$k$  coefficient for non-linearly distributed internal stresses  
1.0 ( $h \leq 300$  mm)... 0.65 ( $h \geq 800$  mm)

$h$ : web height or flange width

NA-D: lower value of the partial cross section  
if internal action applies,  $k \cdot 0.8$

$f_{\text{ct,eff}}$  tensile strength,  $f_{\text{ctm}}$  ( $t \leq 28d$ )  
NA-D:  $\geq 2.9$  N/mm<sup>2</sup> when  $t \geq 28$  d

$k_c$  coefficient for the stress distribution

$$k_c = 0.4 \cdot (1 - \sigma_c / (k_1 \cdot f_{\text{ct,eff}} \cdot h/h'))$$

$\sigma_c$ : concrete stress (state I) under internal crack forces  
in the centre of gravity of the partial cross section

Flanges hollow box, T-cross sections, for internal crack forces completely under tension

$$k_c = 0.9 \cdot F_{\text{cr}} / (A_{\text{ct}} \cdot f_{\text{ct,eff}}) \geq 0.5$$

$F_{\text{cr}}$ : tensile force in the flange under internal crack forces (state I)

$\sigma_s$ : Tab. 7.2N with  $D_{s1}$ , derivation see /54/ p. 7-6

$$D_{s1} = D_s \cdot f_{\text{ct0}} / f_{\text{ct,eff}} \cdot 2 \cdot (h-d) / (k_c \cdot h_{\text{cr}})$$

**NA-D, NA-A:**

As is calculated directly if  $F_s = F_{cr} = k \cdot k_c \cdot f_{cteff} \cdot A_{ct}$ .

$$F_{cr} < F_{cre} = A_{ceff} \cdot f_{cteff}$$

$$A_s = \sqrt{\frac{d_s \cdot (1 - \beta_t) \cdot F_s \cdot F_s}{3.6 \cdot E_s \cdot w_k \cdot f_{cteff}}}$$

Otherwise

$$A_s = \sqrt{\frac{d_s \cdot F_{cre} \cdot (F_s - \beta_t \cdot F_{cre})}{3.6 \cdot E_s \cdot w_k \cdot f_{cteff}}}$$

## Strain verification in accordance with EN 1992-1-1

### Concrete, infrequent combination

$$\sigma_c < k_1 \cdot f_{ck}$$

The objective is to prevent the destruction of the concrete structure. Alternatively, you can increase the concrete cover or enclose the compression zone with reinforcement.

### Concrete, quasi-permanent combination

$$\sigma_c < k_2 \cdot f_{ck}$$

When this limit value is exceeded, linear creep can no longer be assumed. If applicable, an increased creep coefficient according to equation 3.7 should be considered.

### Reinforcing steel, infrequent combination

$$\sigma_s < k_3 \cdot f_{yk}$$

Whereas the crack width verification for reinforced concrete is performed for the quasi-permanent combination, yielding of the reinforcement should also be prevented if the infrequent combination applies.

With indirect action:  $\sigma_s < k_4 \cdot f_{yk}$

	k1	k2	k3	k4	Comment
EN	0.6	0.45	0.8	1.0	k1: recommended with the exposure classes XD, XS or XF.
NA-D	= EN	= EN	= EN	= EN	k1: can be dispensed with where unprestressed components in typical building construction are concerned if the percentage of the redistribution is < 15 %.
NA-GB	= EN	= EN	= EN	= EN	
NA-A	= EN	= EN	= EN	= EN	
NA-I	= EN	= EN	= EN	= EN	k1: reduced by 20 % if h ≤ 50 mm
NA-PL	= EN	= EN	= EN	= EN	

### Calculation of the existing stresses

In accordance with /11/, the steel stresses should be calculated with a reduced modulus of elasticity  $E_{\text{eff}} = E_{\text{cm}} / (1 + \varphi(t_0, \infty))$ .

This calculation method takes the long-term behaviour of concrete into account. The concrete withdraws from its participation in load bearing by creep i.e. by redistribution to the reinforcing steel.

Acc. to /11/, this can often be neglected where compact cross sections are concerned. With T-beams, however, the resultant steel stresses increase by 5 % in comparison to a calculation that does not consider the creep coefficient. A corresponding note as in ENV 1992 1-1 Para. 4.4.1.3 (3) is however missing in EN 1992 1-1.

Correspondingly, early points in time are decisive for the calculation of the concrete stresses, i.e.  $\varphi = 0$  in this case.

### NA-A:

Reinforcing steel stresses with the accidental load combination:

$$\text{Equation: } \varphi_{\text{eff}}(t_0, \infty) = \varphi(t_0, \infty) \cdot \frac{M_{\text{qp},k}}{M_{\text{E0},k}}$$

$M_{\text{qp},k}$ : bending moment with the quasi-permanent load combination

$M_{\text{E0},k}$ : bending moment with the infrequent load combination

Reinforcing steel stresses with the infrequent load combination:

According to the NA, a calculation with  $\varphi_{\text{eff}}(t_0, t)$  with  $t = \text{start of usage}$  is possible. This option is currently not implemented due to its insignificance.

Concrete stresses in the quasi-permanent load combination:

Unpre-stressed load bearing structures always with  $\varphi(t_0, \infty)$ .

This assumption is implemented as default in B2.

## Accidental design situation fire

The design or the calculation of the stiffness for rectangular and circular cross sections with fire exposure on 1, 3 and 4 sides is implemented. (Note: B5 currently only 4-sides).

### Fundamental considerations

The verification is performed in accordance with the requirements applying to a general calculation method. It includes a FEM-based temperature analysis with the parameters defined in the National Annexes (TA module is required) and a mechanical analysis to determine the internal forces with the help of the stress-strain curves of concrete and steel of EN 1992-1-1 and the determination of the balance with the external forces with consideration to thermal strain.

#### B2 application - reinforced concrete design

As the exact location and position of the steel is decisive for the result, the additional module "Polygonal design" B2-Poly should be available. The verifications under fire exposure are performed for the cross section types "rectangle with general point reinforcement" and "circle with general point reinforcement".

If the TA add-on module is not available, temperatures can be assessed by approximation with the help of the diagrams in EN 1992-1-2 Annex A. In this case, results may be non-compliant with the assumptions specified in some National Annexes, however.

#### Border conditions for the temperature analysis in the various National Annexes

	Component moisture %	Density $\rho$ [kg/m <sup>3</sup> ]	Conductivity $\lambda$ as per NA
EN (Annex A)	1.5	2300	$\lambda_u$
NA-D	3	2400	$\lambda_o$
NA-A	= EN	= EN	= EN High strength: $\lambda_o$
NA-GB	= EN	= EN	= EN High strength: $\lambda_o$
NA-PL	= EN	= EN	= EN

*Note: Component moisture and density are no NDPs. In Germany, these parameters do not comply with the assumptions stated by EN 1992-1-2 Annex A, however. See approximation method as per DIN EN 1992-1-2/NA Annex AA for instance.*

### External forces

Forces of the combination for the accidental design situation fire should be used in accordance with EN 1990. In contrast to EN 1990, EN 1991-1-2 allows the use of a quasi-permanent value of  $\psi_{2,1} \cdot Q_{k,1}$  for the decisive variable action.

(NA-D: not allowed if wind is the leading action).

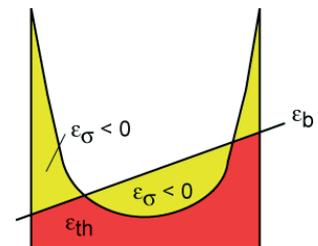
## Internal forces

In order to calculate the internal forces acting on the concrete, the concrete cross section is divided into elements with an edge length of 1 cm each. The internal forces of the element result with the stress-strain curves corresponding to the average element temperatures acc. to /42/ figure 3.1 and table 3.1. Calcereous aggregates can be taken into account, if applicable. The thermal strain results according to figure 3.5. For high-strength concretes, modified stress strain curves as per table 6.1 N are used (NA-A: table 1):

The internal forces on the reinforcing steel depend on the temperatures in the reinforcement points acc. to /42/ figure 3.3 and table 3.2. The more favourable behaviour of hot-rolled steel can be taken into account in this connection, if applicable. According to /44/, steel of class X requires a proof by experimental testing and is therefore currently not supported. The thermal strain results according to /42/ figure 3.

The stress-generating strain  $\varepsilon_{\sigma}$  in an arbitrary point of the cross section results from the thermal strain  $\varepsilon_{th}$  depending on the temperature and the bending strain  $\varepsilon_b$  in this point. The equation  $\varepsilon_{\sigma} = \varepsilon_b - \varepsilon_{th}$  applies.

A typical bearing behaviour results for the concrete, whereby a smaller outer ring due to the considerably diminished stress-strain curve at high temperatures and an inner area with  $\varepsilon_{\sigma} > 0$  (tension) withdraw from their participation in the bearing of the loads.



The internal forces on the reinforcing steel react quite sensibly to the location of the reinforcement point, a minor change in position of 1 cm produces a measurable change in the steel strain.

The internal forces acting on the steel are calculated with consideration to the individual rebars. The effective stiffness results from the found strain state.

## Design

The strain state (bending plane) at which the internal and external forces are in balance is sought after by iterative approximation.

The internal forces on the steel are first calculated for a reinforcement area still unknown whereby a uniform weighting of the entered reinforcement points is assumed.

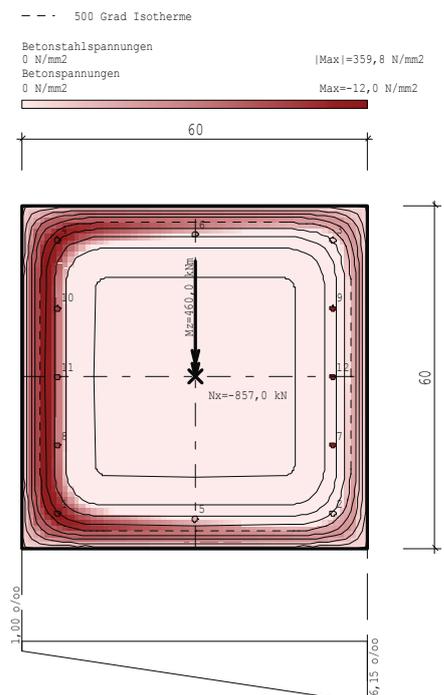
The strain plane is varied between the defined breakage strains. The required reinforcement quantity results directly from the resultant strain state.

## Calculation of the effective stiffness

→ See [Calculation of the effective stiffness](#).

## Validation examples

According to DIN EN 1991-1-2/NA, the software applications used for the general verification method should be validated with the help of the examples specified in Annex CC. Validation examples within the verification range of B2 are CC4.8 and CC4.9 - weakly and strongly reinforced beams.



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